Gov 50: 20. Hypothesis testing

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Roadmap

1. The lady tasting tea
2. Hypothesis tests
3. Hypothesis testing using infer

1/ The lady tasting tea

The lady tasting tea

Your friend asks you to grab a tea with milk for her before meeting up and she says that she prefers tea poured before the milk. You stop by a local tea shop and ask for a tea with milk. When you bring it to her, she complains that it was prepared milk-first.

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  - Prepare 8 cups of tea, 4 milk-first, 4 tea-first
  - Present cups to friend in a **random** order
  - Ask friend to pick which 4 of the 8 were milk-first.

Lady Tasting Tea data

Friend picks out all 4 milk-first cups correctly!

```r
library(gov50data)
tea
```

```
## # A tibble: 8 x 2
##   truth      guess
##   <chr>      <chr>
## 1 tea-first  tea-first
## 2 milk-first milk-first
## 3 milk-first milk-first
## 4 tea-first  tea-first
## 5 tea-first  tea-first
## 6 milk-first milk-first
## 7 tea-first  tea-first
## 8 milk-first milk-first
```

Thought experiment

Could she have been guessing at random? What would guessing look like?

```r
set.seed(02138)
one_guess <- tea |>
  mutate(random_guess = sample(guess))
one_guess
```

```
## # A tibble: 8 x 3
##   truth      guess      random_guess
##   <chr>      <chr>      <chr>
## 1 tea-first  tea-first  milk-first
## 2 milk-first milk-first tea-first
## 3 milk-first milk-first tea-first
## 4 tea-first  tea-first  milk-first
## 5 tea-first  tea-first  tea-first
## 6 milk-first milk-first milk-first
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```

4 correct in this random guess!

Another guess

```r
another_guess <- tea |>
  mutate(random_guess = sample(guess))
another_guess
```

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## # A tibble: 8 x 3
##   truth      guess      random_guess
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6 correct in this random guess!

All possible guesses

We could enumerate all possible guesses. “Guessing” would mean choosing one of these at random:

```
##   Cup 1 Cup 2 Cup 3 Cup 4 Cup 5 Cup 6 Cup 7 Cup 8
## 1  milk  milk  milk  milk   tea   tea   tea   tea
## 2  milk  milk  milk   tea  milk   tea   tea   tea
## 3  milk  milk   tea  milk  milk   tea   tea   tea
## 4  milk   tea  milk  milk  milk   tea   tea   tea
## 5   tea  milk  milk  milk  milk   tea   tea   tea
## 6  milk  milk  milk   tea   tea  milk   tea   tea
```

[snip]

```
##    Cup 1 Cup 2 Cup 3 Cup 4 Cup 5 Cup 6 Cup 7 Cup 8
## 65   tea   tea   tea  milk  milk   tea  milk  milk
## 66  milk   tea   tea   tea   tea  milk  milk  milk
## 67   tea  milk   tea   tea   tea  milk  milk  milk
## 68   tea   tea  milk   tea   tea  milk  milk  milk
## 69   tea   tea   tea  milk   tea  milk  milk  milk
## 70   tea   tea   tea   tea  milk  milk  milk  milk
```

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  - Impossible? No, because of random chance!

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- Example 2:
  - Trump won 47.5% of the vote in the 2020 election.
  - Last YouGov poll of 1,363 likely voters said 44% planned to vote for Trump.
  - Could the difference between the poll and the outcome be just due to random chance?

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- **Probabilistic** proof by contradiction: try to “disprove” the null.

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- Data: poll has $\overline{X} = 0.44$ with $n = 1363$.

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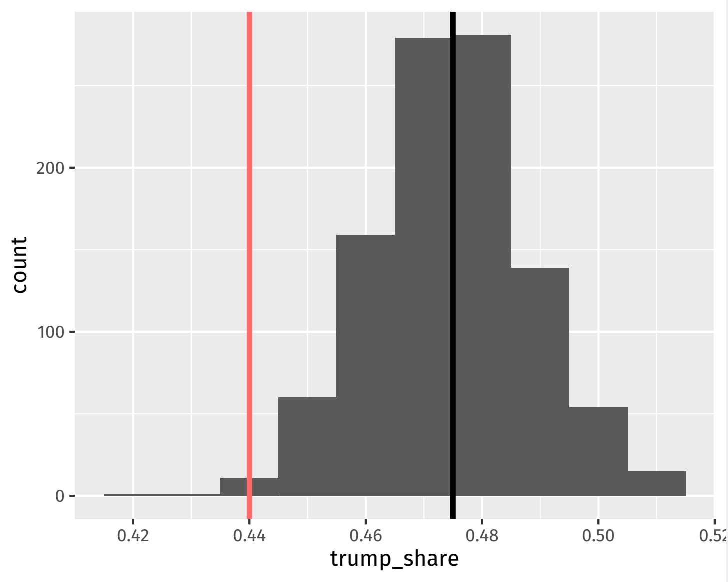
- Compare the distribution of proportions under the null to the observed proportion.

```
null_dist <- tibble(
  trump_share = rbinom(n = 1000, size = 1363, prob = 0.475) / 1363
)
ggplot(null_dist, aes(x = trump_share)) +
  geom_histogram(binwidth = 0.01) +
  geom_vline(xintercept = 0.44, color = "indianred1", size = 1.25) +
  geom_vline(xintercept = 0.475, size = 1.25)
```

Simulations of the reference distribution



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  - Probability of sample proportions being greater than 0.475 + 0.035 = 0.51.

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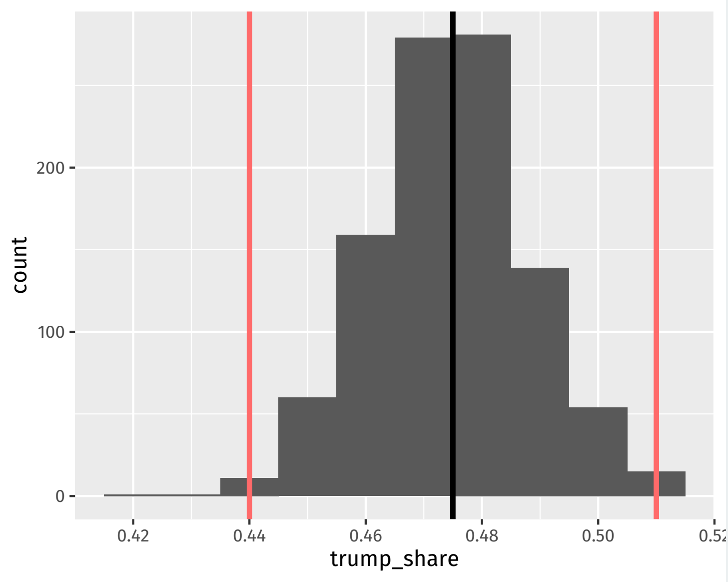
  - Probability of sample proportions being greater than 0.475 + 0.035 = 0.51.

```
mean(null_dist$trump_share < 0.44) + mean(null_dist$trump_share > 0.51)
```

```
## [1] 0.01
```

Two-sided p-value



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    - $p \geq 0.1$ "not statistically significant"

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- Tests usually end with a decision to reject the null or not.
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  - **Test level** α: the threshold for a test.
  - Decision rule: "reject the null if the p-value is below α"
  - Otherwise "fail to reject" or "retain", not "accept the null"
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    - "Convicting" an innocent null hypothesis
- Type II error less serious
    - Missed out on an awesome finding

3/ Hypothesis testing using infer

GSS data from `infer`

```r
library(infer)
gss
```

```
## # A tibble: 500 x 11
##     year   age sex    college   partyid hompop hours income
##    <dbl> <dbl> <fct>  <fct>     <fct>    <dbl> <dbl> <ord>
##  1  2014    36 male   degree    ind          3    50 $25000~
##  2  1994    34 female no degree rep          4    31 $20000~
##  3  1998    24 male   degree    ind          1    40 $25000~
##  4  1996    42 male   no degree ind          4    40 $25000~
##  5  1994    31 male   degree    rep          2    40 $25000~
##  6  1996    32 female no degree rep          4    53 $25000~
##  7  1990    48 female no degree dem          2    32 $25000~
##  8  2016    36 female degree    ind          1    20 $25000~
##  9  2000    30 female degree    rep          5    40 $25000~
## 10  1998    33 female no degree dem          2    40 $15000~
## # ... with 490 more rows, and 3 more variables:
## #   class <fct>, finrela <fct>, weight <dbl>
```

What is the average hours worked?

`dplyr` way:

```
gss |>
  summarize(mean(hours))
```

```
## # A tibble: 1 x 1
##   `mean(hours)`
##           <dbl>
## 1          41.4
```

`infer` way:

```
observed_mean <- gss |>
  specify(response = hours) |>
  calculate(stat = "mean")
observed_mean
```

```
## Response: hours (numeric)
## # A tibble: 1 x 1
##    stat
##   <dbl>
## 1  41.4
```

Hypothesis test

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$$H_1: \mu_{\texttt{hours}} \neq 40$$

How do we perform this test using infer? The **bootstrap!**

Specifying the hypotheses

```
gss |>
  specify(response = hours) |>
  hypothesize(null = "point", mu = 40)
```

```
## Response: hours (numeric)
## Null Hypothesis: point
## # A tibble: 500 x 1
##    hours
##    <dbl>
##  1    50
##  2    31
##  3    40
##  4    40
##  5    40
##  6    53
##  7    32
##  8    20
##  9    40
## 10    40
## # ... with 490 more rows
```

Generating the null distribution

We can use the bootstrap to determine how much variation there will be around 40 in the null distribution.

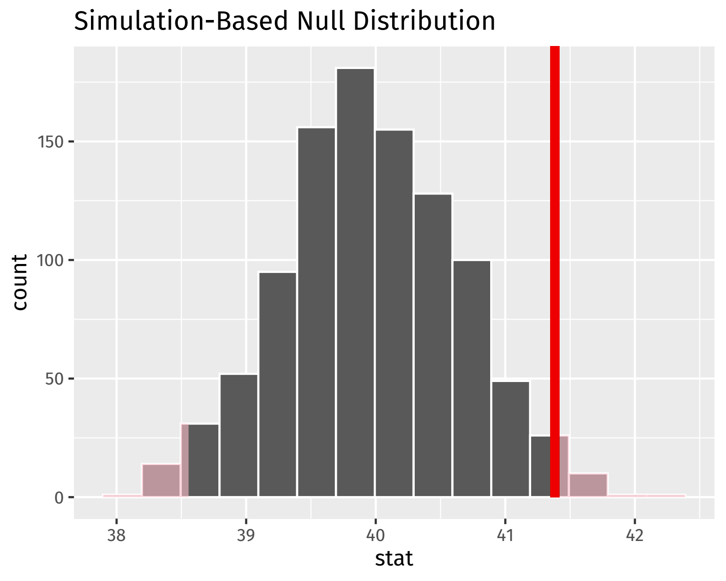
```r
null_dist <- gss |>
  specify(response = hours) |>
  hypothesize(null = "point", mu = 40) |>
  generate(reps = 1000, type = "bootstrap") |>
  calculate(stat = "mean")
null_dist
```

```
## Response: hours (numeric)
## Null Hypothesis: point
## # A tibble: 1,000 x 2
##    replicate  stat
##        <int> <dbl>
##  1         1  40.3
##  2         2  39.8
##  3         3  40.0
##  4         4  39.2
##  5         5  40.3
##  6         6  40.2
##  7         7  40.4
```

Visualizing the p-value

We can visualize our bootstrapped null distribution and the p-value as a shaded region:

```
null_dist |>
  visualize() +
  shade_p_value(observed_mean,
                direction = "two-sided")
```


Simulation-Based Null Distribution
count
150
100
50
0
38
39
40
41
42
stat